Final exam
Electronics & Signal processing
11-04-2017
Prof. Dr. G. Palasantzas

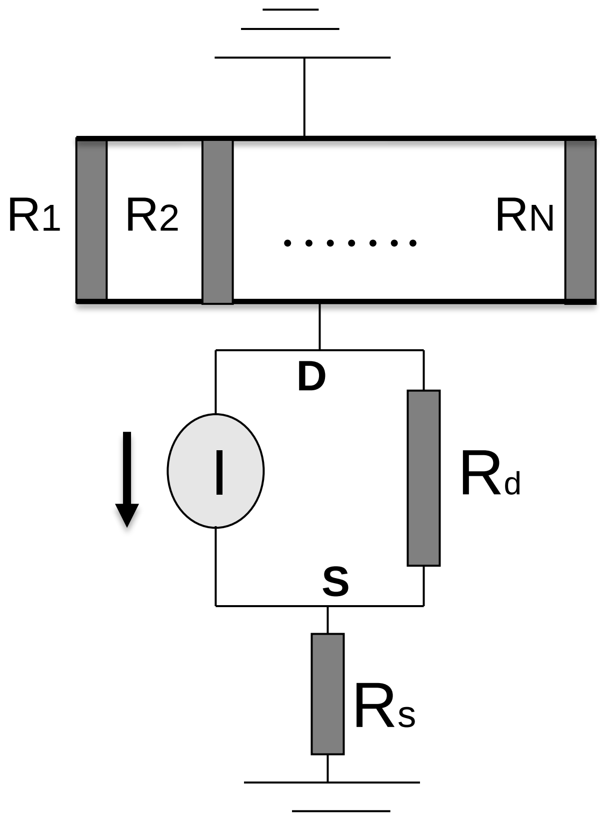
Grade of written exam:

Mark is cummulative points scored for all problems

Total maximum score : 10

Problem 1 (1.5 points)

Consider the current source I in the circuit shown below to be parallel with a resistor R_d

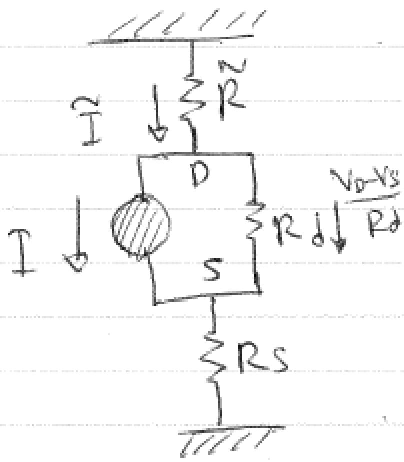


(a: 1 point) Calculate the potential at point D

(b: 1/2 points) Calculate the current through each resistance Ri, i={1,N}

Solution-Problem1

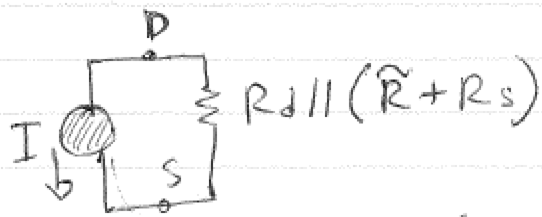
$$\tilde{R} = R_1 \| R_2 \| \cdots \| R_N, \quad \frac{1}{\tilde{R}} = \sum_{i=1}^{N} \frac{1}{R_i}$$



$$\tilde{I} = I + \frac{V_D - V_S}{R_d} \quad (1)$$

$\tilde{R} + R_s$ are parallel to R_d

Thus we have:



$$V_S - V_D = I \cdot R_d \| (\tilde{R} + R_s) \quad (2)$$

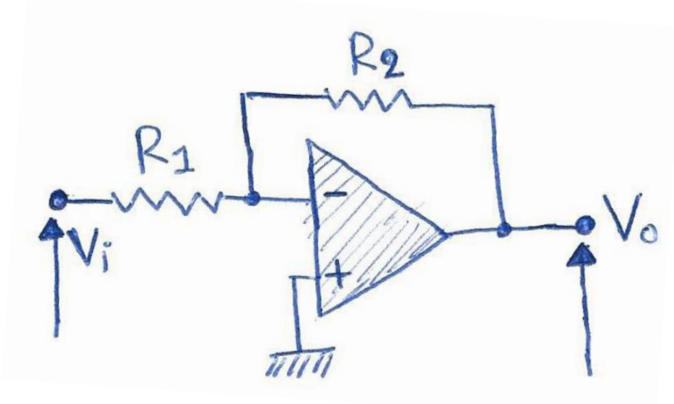
$$(1) \;\&\; (2) \Rightarrow \tilde{I} = I \left\{ 1 - \frac{R_d \| (\tilde{R} + R_s)}{R_d} \right\} \quad (3)$$

$$V_D = -\tilde{I} \cdot \tilde{R} \Rightarrow \underbrace{V_D = -I \left\{ \tilde{R} - \frac{\tilde{R}}{R_d} * \left[R_d \| (\tilde{R} + R_s) \right] \right\}}_{(a)}$$

$$I_i = -\frac{V_D}{R_i} \Rightarrow \underbrace{I_i = +\frac{I}{R_i} \left\{ \tilde{R} - \frac{\tilde{R}}{R_d} * \left[R_d \| (\tilde{R} + R_s) \right] \right\}}_{(b)}$$

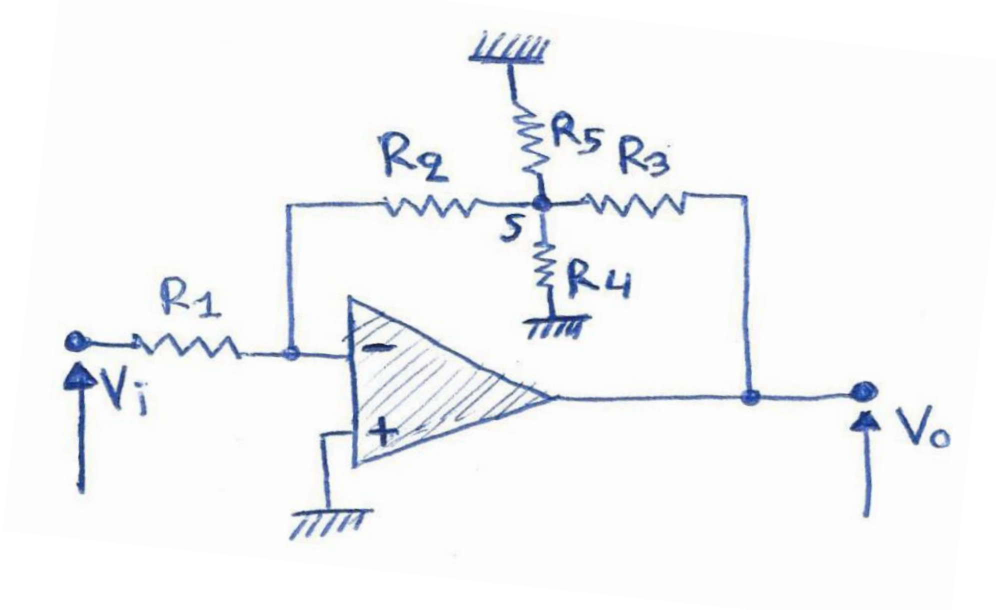
Problem 2 (2.5 points)

(a: 1 point) Consider the opamp to have infinite input and zero output resistance, but finite forward open loop gain A so that Vo=A(V+ - V-)



(a1: 1/2 points) Calculate the closed loop gain Vo/Vi
(a2:: 1/2 points) Calculate the limit of Vo/Vi for an ideal opamp with $A \to +\infty$

(b: 1.5 points) Consider the opamp to be ideal with infinite input and zero output resistance, and infinite forward open loop gain ($A=+\infty$) so that V+=V-

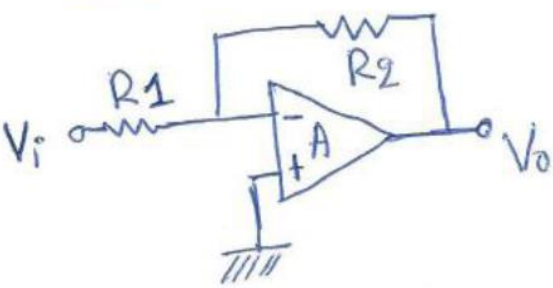


(b1: 1/2 points) Calculate the potential at point S
(b2: 1 point) Calculate the closed loop gain Vo/Vi

Solution-problem-2

(a)

Negative feedback



$$V_o = A\,(V_+ - V_-) = -A V_- \quad (1)$$

$$V_- = V_i \frac{R_2}{R_1+R_2} + V_o \frac{R_1}{R_1+R_2} \quad (2)$$

$$(1) \Rightarrow V_- = -V_o/A$$

$$(2) \Rightarrow \quad V_o\left(1 + A\frac{R_1}{R_1+R_2}\right) = -V_i \frac{R_2}{R_1+R_2} A \quad (3)$$

(a1) $$V_o / V_i = -\frac{R_2}{R_1+R_2} A \Big/ \left(1 + A\frac{R_1}{R_1+R_2}\right)$$

(a2) $$V_o / V_i = -\frac{R_2}{R_1}$$

$$A \rightarrow \infty$$

(b)

$V_+ = V_- = 0$

R_5, R_4, R_2 are parallel if

we define $\tilde{R} = R_2 // R_4 // R_5$, then using a voltage divider between $\tilde{R}$ and R_3 we have

$$\boxed{V_S = V_o \cdot \frac{\tilde{R}}{\tilde{R} + R_3}} \quad (b1)$$

The same current flows in R_1 and R_2

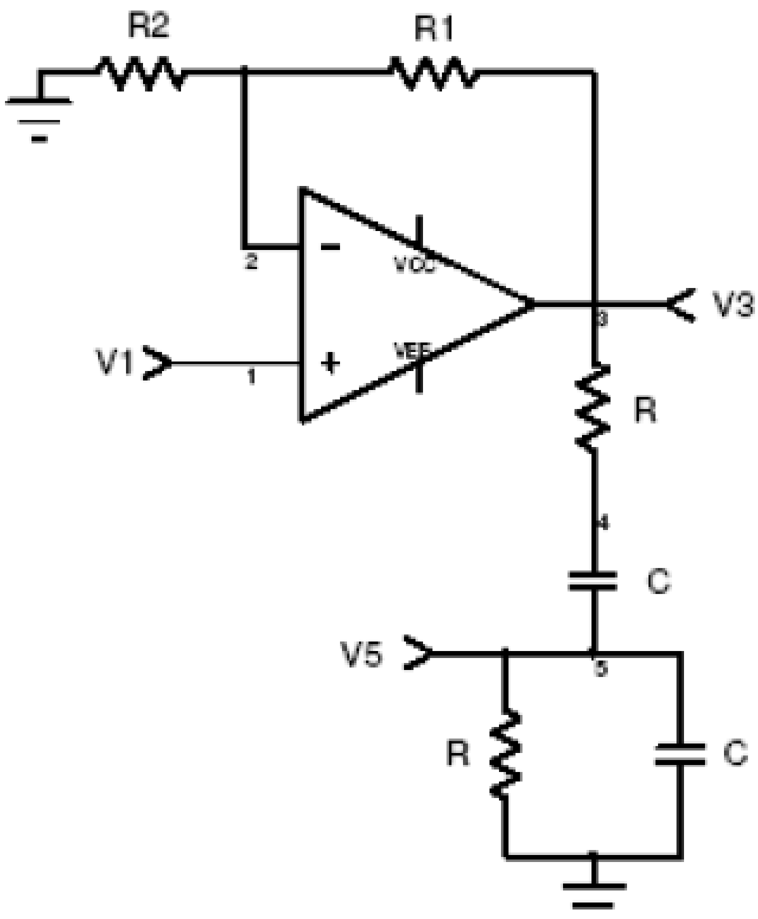
So we have $\frac{V_i - 0}{R_1} = \frac{0 - V_S}{R_2} \Rightarrow$

$$\frac{V_i}{R_1} = -\frac{V_S}{R_2} = -\frac{\tilde{R}}{R_2} \cdot \frac{1}{R_3 + \tilde{R}} \cdot V_o \Rightarrow$$

$$\boxed{\frac{V_o}{V_i} = -\frac{(R_3 + \tilde{R}) R_2}{\tilde{R} R_1}} \quad (b2)$$

Problem 3 (1.5 points)

Consider the oscillating circuit (Wien Oscillator) with $V_+ = V_-$



(a: 1/2 points) Calculate the transfer A = V(3) / V(1)

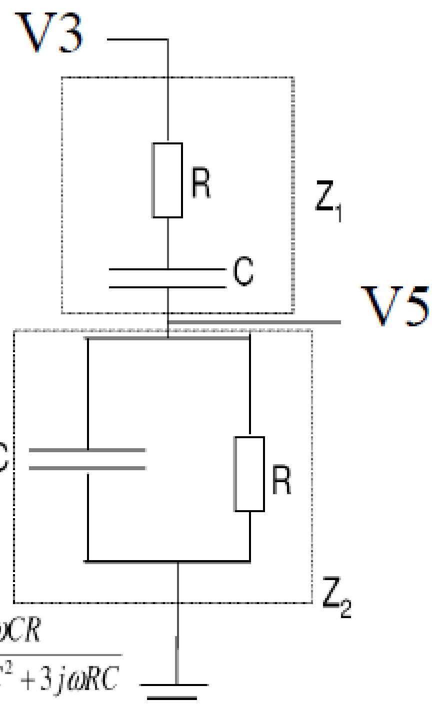
(b: 1/2 points) Calculate the transfer B = V(5) / V(3). For what value of ωRC is B real?

(c: 1/2 points) For what value of R_1 / R_2 is AB = 1? Note that now V(5) = V(1).

Solution-problem-3

(a) V+=V1, V-=V3{R2/[R2+R1]} (voltage divider)
V+=V- → V1= V3{R2/R2+R1} → A=V3/V1=1+(R1/R2)

(b) In the more general case we have:



$$\frac{V5}{V3} = \frac{Z_2}{Z_1+Z_2}$$

$$Z_1 = R + \frac{1}{j\omega C} = \frac{j\omega RC+1}{j\omega C}$$

$$\frac{1}{Z_2} = \frac{1}{R} + j\omega C = \frac{1+j\omega RC}{R} \rightarrow Z_2 = \frac{R}{1+j\omega RC}$$

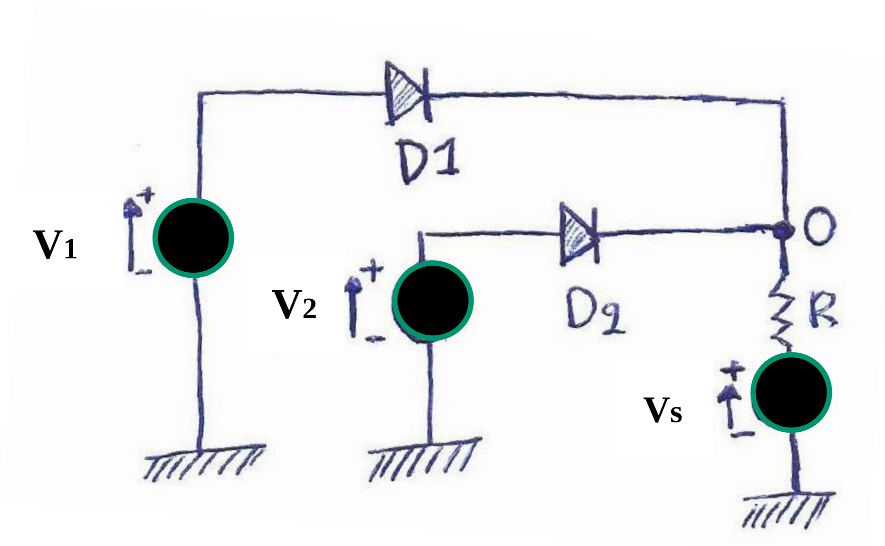
$$\frac{V5}{V3} = \frac{\dfrac{R}{1+j\omega RC}}{\dfrac{j\omega RC+1}{j\omega C} + \dfrac{R}{1+j\omega RC}} = \frac{R}{\dfrac{1+2j\omega RC-\omega^2R^2C^2}{j\omega C}+R} = \frac{j\omega CR}{1-\omega^2R^2C^2+3j\omega RC}$$

$$= \frac{1}{\dfrac{1-\omega^2R^2C^2}{j\omega RC}+3} = \frac{1}{3-j\dfrac{1-\omega^2R^2C^2}{\omega RC}}$$

For ω=1/RC B=V5/v3 is real → B=1/3

(c) AB=1, since B=1/3→ A=3 → R1/R2=2
Under these condition we have formed the Wien Oscillator

Problem 4 (1.5 points)



(a: 1/2 point) The diodes D1 and D2 are ideal with forward conduction voltage Vc=0.7 V. Explain briefly which diode conducts current, and justify your answer. Consider V_1=4 V, V_2=9 V, and V_s=2 V

(b: 1/2 point) Calculate the current via the resistor R.

(c: 1/2 points) How much power is consumed on the diode that conducts current?

Solution-problem-4

(a)

For diode D1 we have 4-2=2>Vc, so in principle it can conduct current
For diode D2 we have 9-2=7>Vc, so in principle it can conduct current.

When D2 is conducting then the potential at the point O is Vo=9-Vc=8.3> 4 Volts. Therefore, the diode D1 will not conduct, and only D2 is conducting!

(b) $I = (V_o-V_s)/R = [(V_2-V_c)-V_s]/R = 6.3/R$

(c) $P_{D2}=I\cdot V_c=(6.3\ V_c)/R=4.41/R$

Problem 5 (1.5 points)

Design a synchronous 4-counter using J-K flip-flops that counts through the states 1, 2, 3, 4:

	Before state			After state		
	Q3	Q2	Q1	Q3	Q2	Q1
1	0	0	1			
2	0	1	0			
3	0	1	1			
4	1	0	0			

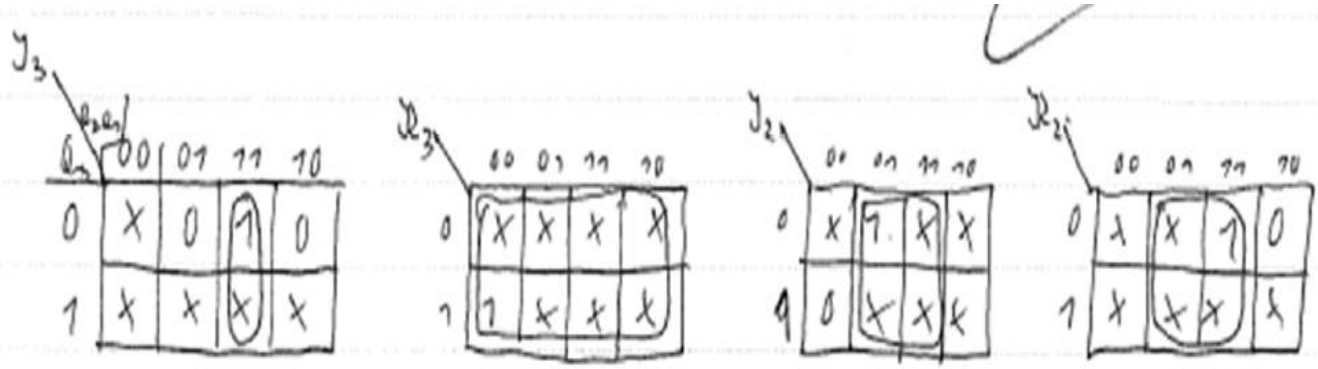
Q_{n-1}	Q_n	J	K
0	0	0	*
0	1	1	*
1	0	*	1
1	1	*	0

*: don't care

J	K	Q_n
0	0	Q_{n-1}
0	1	0
1	0	1
1	1	$\overline{Q_{n-1}}$

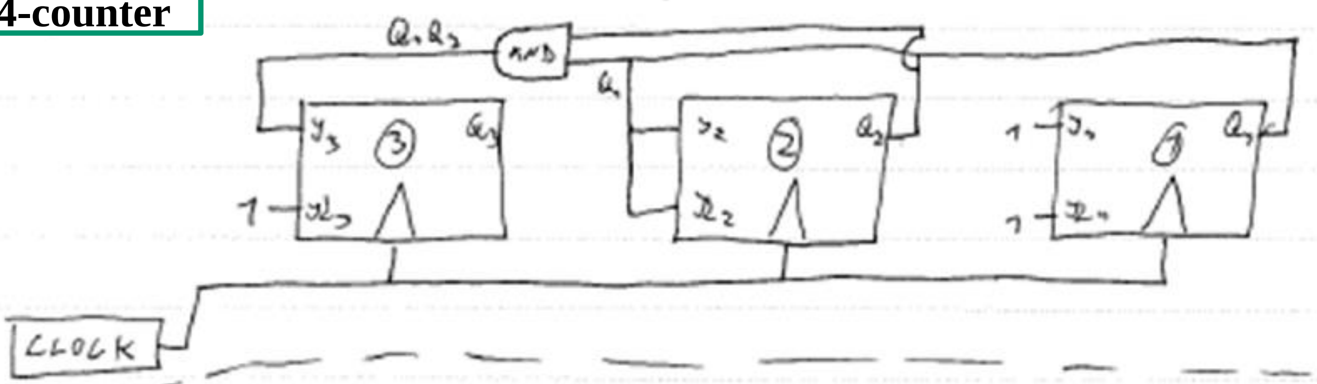
Solution-problem-5

	Q_3	Q_2	Q_1	Q_3	Q_2	Q_1	J_3	K_3	J_2	K_2	J_1	K_1
1	0	0	1	0	1	0	0	x	1	x	x	1
2	0	1	0	0	1	1	0	x	x	0	1	x
3	0	1	1	1	0	0	1	x	x	1	x	1
4	1	0	0	0	0	1	x	1	0	x	1	x



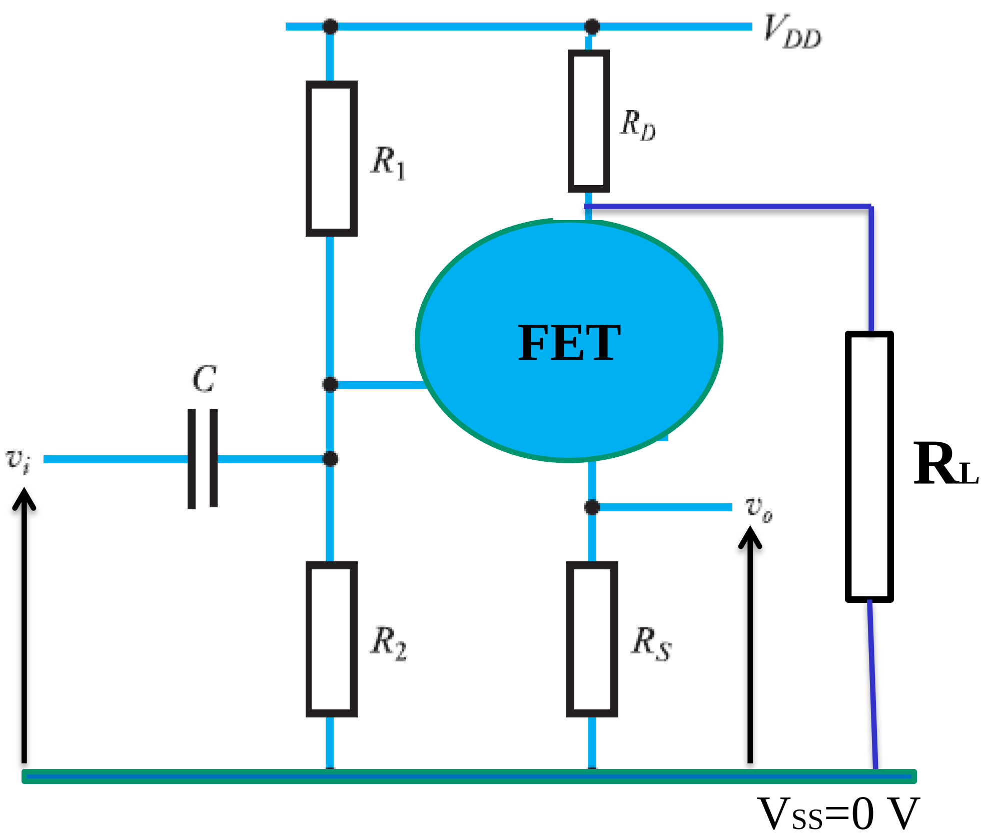
J2=K2=Q1, J3=Q2Q1 and K3=1 J1=K1=1

4-counter



Problem 6 (1.5 points)

Consider a FET as shown bellow:



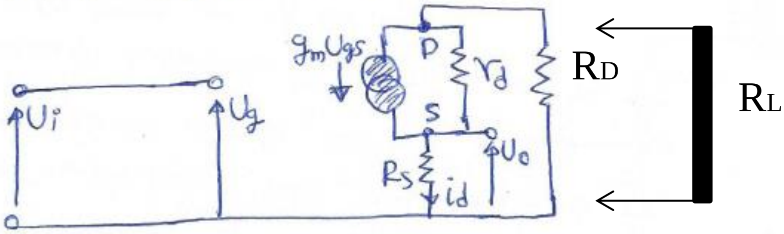
Show that the amplification ratio υ_o / υ_i of the small voltage variation at the input/output is given by:

$$\frac{\upsilon_o}{\upsilon_i} = \frac{g_m R_s}{1 + g_m R_S + [(R_{DL} + R_S)/r_d]}, \quad R_{DL} = R_D \,//\, R_L$$

with g_m the transconductance, and r_d the differential resistance of the FET operating at saturation.

Solution-problem-6

Small signal circuit



(1) $V_o = V_s = i_d R_s$, $i_b = g_m V_{gs} + \dfrac{V_d - V_s}{r_d}$ (2)

$i_b = -\dfrac{V_d}{R_D} = \dfrac{V_s}{R_S} \Rightarrow V_d = -\dfrac{R_D}{R_S} V_s$ (3)

(1) & (2) & (3) $\Rightarrow$

$$V_s = R_s\left[g_m V_g - g_m V_s - \frac{1+\frac{R_D}{R_S}}{r_d} V_s\right]$$

$$\Rightarrow V_s\left[1 + g_m R_s + \frac{R_D + R_S}{r_d}\right] = g_m R_s V_g$$

$V_s = V_o$, $V_i = V_o$

So we obtain

$$\boxed{\frac{V_o}{V_i} = \frac{g_m R_S}{1 + R_S g_m + \frac{R_D + R_S}{r_d}}}$$

Since RD are RL are parallel when you make the small signal circuit then just replace RD→ RDL and you have the solution you need

$$\frac{\upsilon_o}{\upsilon_i} = \frac{g_m R_s}{1 + g_m R_S + [(R_{DL} + R_S)/r_d]}, \quad R_{DL} = R_D \,//\, R_L$$

Alternative solution only for the <u>absolute top tough cookie students</u>

<u>Or you do it or you do not do it in this way !</u>

(1) $V_D = V_{DD} - \tilde{I}_D \cdot R_D$, $\tilde{I}_D = I_D + I_L$

(2) $V_D + v_d = V_{DD} - \tilde{I}'_D \cdot R_D$ $\quad \tilde{I}'_D = (I_D + i_d) + (I_L + i_L)$

(2) − (1) $\Rightarrow$ $v_d = -(i_d + i_L) \cdot R_D$ (3)

$V_D = I_L \cdot R_L$ & $V_D + v_d = (i_L + I_L) R_L \Rightarrow$

$v_d = i_L \cdot R_L \Rightarrow$

$i_L = \frac{v_d}{R_L}$ (4) , $i_d = g_m v_{gs} + \frac{v_d - v_s}{r_d}$ (5)

(4) + (5) in (3) $\Rightarrow$

$$v_d = -\left[g_m v_{gs} + \frac{v_d - v_s}{r_d}\right] R_D - v_d \frac{R_D}{R_L} \quad (6)$$

$$\frac{v_d}{R_{DL}} \cancel{R_D} = -\cancel{R_D} g_m v_{gs} - \frac{v_d - v_s}{r_d} \cancel{R_D} \qquad \underbrace{\frac{1}{R_{DL}} = \frac{1}{R_D} + \frac{1}{R_L}}$$

$$\frac{v_d}{R_{DL}} = -g_m v_{gs} - \frac{v_d - v_s}{r_d} \quad (7)$$

$\tilde{I}_D = I_D + I_L$ $\quad\quad$ $\tilde{I}_D' = (I_D + i_d) + (I_L + i_L)$

$$\frac{V_{DD} - V_D}{R_D} = \frac{V_S}{R_S} + \frac{V_D}{R_L}, \quad \frac{V_{DD} - V_D - V_d}{R_D} = \frac{V_S + V_s}{R_S} + \frac{V_D + V_d}{R_L}$$

$\Rightarrow$ substraction gives $\boxed{-\frac{V_d}{R_{DL}} = +\frac{V_s}{R_S} \quad (8)}$ $\quad$ $\boxed{\frac{1}{R_{DL}} = \frac{1}{R_D} + \frac{1}{R_L}}$

From (7) & (8) we eliminate V_d and we set $V_g = V_i$

$$-\frac{V_s}{R_S} = -g_m V_i + g_m V_s + \frac{R_{DL} + R_S}{R_S r_d} V_s \Rightarrow$$

$$V_s = g_m R_S V_i - g_m R_S V_s - \frac{R_{DL} + R_S}{r_d} V_s \Rightarrow$$

$$\boxed{\frac{V_s}{V_i} = \frac{V_o}{V_i} = \frac{g_m R_S}{1 + g_m R_S + \frac{R_{DL} + R_S}{r_d}}}$$